

# A General Factoring Strategy

## 0. **Order:**

Order the terms in descending order according to the powers of the variable of your choice.

## 1. **GCF:**

Factor out a Greatest Common Factor if possible. Set it aside and be sure to include it in the final answer.

## 2. **Special Cases:**

If there are two terms, then consider the following cases:

(a)  $A^2 - B^2 = (A - B)(A + B)$

(b)  $A^2 + B^2 = \text{PRIME}$

(c)  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

(d)  $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

## 3. **Factor by Grouping, Part I:**

If there are three terms,

(a) multiply the coefficient of the first term by the coefficient of the last term

(b) factor this result into all possible pairs of factors

(c) add or subtract each pair of factors until you find a combination that produces the middle term

(d) replace the middle term with that sum or difference so that each piece is multiplied by the variable(s)

(Note: you should now have four terms of which the middle two are like each other and the previous middle term.)

## 4. **Factor by Grouping, Part II:**

If there are four terms,

(a) factor the first pair

(b) then factor the second pair

(c) if there is a common binomial left in each part after factoring above, then factor this binomial out, and multiply the remaining terms by it in parentheses

(d) if there is not a common binomial, then either you made a mistake or the polynomial cannot be factored this way

**REMEMBER:** If you factored something out in step two, then be sure to include it in your final answer.

### Examples:

1. Factor  $3x^2 + 9x + 6$  completely

$$3x^2 + 9x + 6$$

**Step 0:** already done

$$3(x^2 + 3x + 2)$$

**Step 1:** factor out three

**Step 2:** does not apply

$$x^2 + 3x + 2$$

**Step 3a:** multiply 1 by 2 to get 2

**Step 3b:** factor 2,  $1 \times 2$

**Step 3c:** combine factors,  $1 + 2 = 3$

$$x^2 + 1x + 2x + 2$$

**Step 3d:** replace  $3x$  with  $1x + 2x$

$$x(x + 1) + 2x + 2$$

**Step 4a:** factor the first pair

$$x(\mathbf{x + 1}) + 2(\mathbf{x + 1})$$

**Step 4b:** factor the second pair

$$(\mathbf{x + 1})(x + 2)$$

**Step 4c:** factor out  $(x + 1)$ , leaving  $(x + 2)$

$$3(x + 1)(x + 2)$$

completely factored answer

2. Factor  $9x^2 + 14x - 8$  completely

$$9x^2 + 14x - 8$$

**Step 0:** already done

**Step 1:** no GCF to factor

**Step 2:** does not apply

$$9x^2 + 14x - 8$$

**Step 3a:** multiply 9 by 8 to get 72

**Step 3b:** factor 72,  $1 \times 72$

$$2 \times 36$$

$$3 \times 24$$

$$4 \times 18$$

$$6 \times 12$$

$$8 \times 9$$

**Step 3c:** combine factors,  $18 - 4 = 14$

$$9x^2 + 18x - 4x - 8$$

**Step 3d:** replace  $14x$  with  $+18x - 4x$

$$9x(x + 2) - 4x - 8$$

**Step 4a:** factor the first pair

$$9x(\mathbf{x + 2}) - 4(\mathbf{x + 2})$$

**Step 4b:** factor the second pair

$$(\mathbf{x + 2})(9x - 4)$$

**Step 4c:** factor out  $(x + 2)$ , leaving  $(9x - 4)$

$$(x + 2)(9x - 4)$$

completely factored answer