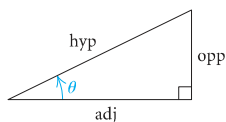


# Trigonometry

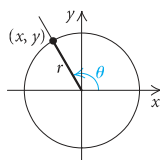
## Trigonometric Functions

### Acute Angles



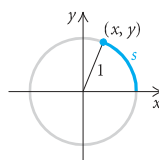
$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}}, & \csc \theta &= \frac{\text{hyp}}{\text{opp}}, \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}}, & \sec \theta &= \frac{\text{hyp}}{\text{adj}}, \\ \tan \theta &= \frac{\text{opp}}{\text{adj}}, & \cot \theta &= \frac{\text{adj}}{\text{opp}}\end{aligned}$$

### Any Angle



$$\begin{aligned}\sin \theta &= \frac{y}{r}, & \csc \theta &= \frac{r}{y}, \\ \cos \theta &= \frac{x}{r}, & \sec \theta &= \frac{r}{x}, \\ \tan \theta &= \frac{y}{x}, & \cot \theta &= \frac{x}{y}\end{aligned}$$

### Real Numbers



$$\begin{aligned}\sin s &= y, & \csc s &= \frac{1}{y}, \\ \cos s &= x, & \sec s &= \frac{1}{x}, \\ \tan s &= \frac{y}{x}, & \cot s &= \frac{x}{y}\end{aligned}$$

## Basic Trigonometric Identities

$$\begin{aligned}\sin(-x) &= -\sin x, \\ \cos(-x) &= \cos x, \\ \tan(-x) &= -\tan x,\end{aligned}$$

$$\begin{aligned}\tan x &= \frac{\sin x}{\cos x}, \\ \cot x &= \frac{\cos x}{\sin x},\end{aligned}$$

$$\begin{aligned}\csc x &= \frac{1}{\sin x}, \\ \sec x &= \frac{1}{\cos x}, \\ \cot x &= \frac{1}{\tan x}\end{aligned}$$

## Pythagorean Identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1, \\ 1 + \cot^2 x &= \csc^2 x, \\ 1 + \tan^2 x &= \sec^2 x\end{aligned}$$

## Identities Involving $\pi/2$

$$\begin{aligned}\sin(\pi/2 - x) &= \cos x, \\ \cos(\pi/2 - x) &= \sin x, & \sin(x \pm \pi/2) &= \pm \cos x, \\ \tan(\pi/2 - x) &= \cot x, & \cos(x \pm \pi/2) &= \mp \sin x\end{aligned}$$

## Sum and Difference Identities

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v, \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v, \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

## Double-Angle Identities

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x, \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1, \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$

## Half-Angle Identities

$$\begin{aligned}\sin \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{2}}, & \cos \frac{x}{2} &= \pm \sqrt{\frac{1 + \cos x}{2}}, \\ \tan \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}\end{aligned}$$

(continued)