

NOTE: It is common to write $(\sin(\theta))^2$ as $\sin^2(\theta)$.

5. $(\sin(\phi) - \cos(\phi))^2$

$$\begin{aligned} &= (\sin(\phi) - \cos(\phi))(\sin(\phi) - \cos(\phi)) \\ &= \sin^2(\phi) - \sin(\phi)\cos(\phi) - \cos(\phi)\sin(\phi) + \cos^2(\phi) \\ &= \sin^2(\phi) - 2\sin(\phi)\cos(\phi) + \cos^2(\phi) \end{aligned}$$

If you reorder the terms, then you get THE Pythagoren Identity:

$$\begin{aligned} &= \sin^2(\phi) + \cos^2(\phi) - 2\sin(\phi)\cos(\phi) \\ &= 1 - 2\sin(\phi)\cos(\phi) \end{aligned}$$

13. $2\cos^2(x) + \cos(x) - 3$

Replace $\cos(x)$ with U :

$$2\cos^2(x) + \cos(x) - 3 = 2U^2 + U - 3$$

This factors as:

$$(2U + 3)(U - 1)$$

Put $\cos(x)$ back in:

$$(2\cos(x) + 3)(\cos(x) - 1)$$

17. $\frac{\sin^2(x)\cos(x)}{\cos^2(x)\sin(x)}$

$$\frac{\sin(x)\sin(x)\cos(x)}{\sin(x)\cos(x)\cos(x)}$$

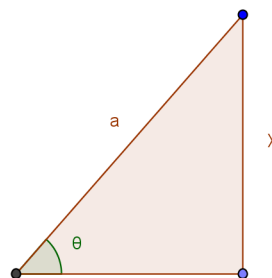
$$\frac{\sin(x)}{\cos(x)} = \tan(x)$$

45. Let $x = a\sin(\theta)$ in $\sqrt{a^2 - x^2}$. Then find $\cos(\theta)$ and $\tan(\theta)$.

The easiest way to do this is to solve $x = a\sin(\theta)$ for $\sin(\theta)$:

$$\sin(\theta) = \frac{x}{a}$$

Draw a right triangle showing this:



Using the Pythagorean theorem, I can find the adjacent side of this triangle:

$$\text{adjacent} = \sqrt{a^2 - x^2}$$

So,

$$\cos(\theta) = \frac{\sqrt{a^2 - x^2}}{a}$$
$$\tan(\theta) = \frac{x}{\sqrt{a^2 - x^2}}$$

53. Find $\tan(105^\circ)$ exactly.

$$\tan(105^\circ) = \tan(60^\circ + 45^\circ) = \frac{\sin(60^\circ + 45^\circ)}{\cos(60^\circ + 45^\circ)}$$

$$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)}$$

I like the second version better,

$$\begin{aligned} &= \frac{\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)} \\ &= \frac{\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2}}{\frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}} \\ &= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{(\sqrt{3} + 1)^2}{-2} \end{aligned}$$