15. $\cos(4x)$

Option 1:

$$\cos(4x) = \cos(2(2x)) = \cos^2(2x) - \sin^2(2x)$$
$$= \left(\cos^2(x) - \sin^2(x)\right)^2 - \left(2\sin(x)\cos(x)\right)^2$$
$$= \cos^4(x) - 2\cos^2(x)\sin^2(x) + \sin^4(x) - 4\sin^2(x)\cos^2(x)$$
$$= \cos^4(x) - 6\cos^2(x)\sin^2(x) + \sin^4(x)$$

Option 2:

$$\cos(4x) = \cos(2(2x)) = \cos^2(2x) - \sin^2(2x)$$
$$= \left(2\cos^2(x) - 1\right)^2 - \left(2\sin(x)\cos(x)\right)^2$$
$$= 4\cos^4(x) - 4\cos^2(x) + 1 - 4\sin^2(x)\cos^2(x)$$

Option 3:

$$\cos(4x) = \cos(2(2x)) = \cos^2(2x) - \sin^2(2x)$$
$$= \left(1 - 2\sin^2(x)\right)^2 - \left(2\sin(x)\cos(x)\right)^2$$
$$= 1 - 4\sin^2(x) + 4\sin^4(x) - 4\sin^2(x)\cos^2(x)$$

Option 4:

$$\cos(4x) = \cos(2(2x)) = 2\cos^2(2x) - 1$$
$$= 2\left(\cos^2(x) - \sin^2(x)\right)^2 - 1$$
$$= 2\cos^4(x) - 4\cos^2(x)\sin^2(x) + 2\sin^4(x) - 1$$

Option 5:

$$\cos(4x) = \cos(2(2x)) = 2\cos^2(2x) - 1$$
$$= 2\left(2\cos^2(x) - 1\right)^2 - 1$$
$$= 8\cos^4(x) - 8\cos^2(x) + 1$$

Option 6:

$$\cos(4x) = \cos(2(2x)) = 2\cos^2(2x) - 1$$
$$= 2\left(1 - 2\sin^2(x)\right)^2 - 1$$
$$= 8\sin^4(x) - 8\sin^2(x) + 1$$

Option 7:

$$\cos(4x) = \cos(2(2x)) = 1 - 2\sin^2(2x)$$
$$= 1 - 2\left(2\sin(x)\cos(x)\right)^2$$
$$= 1 - 8\sin^2(x)\cos^2(x)$$

32. $\dfrac{1+\sin(2x)+\cos(2x)}{1+\sin(2x)-\cos(2x)}$

$$\frac{1+\sin(2x)+\cos(2x)}{1+\sin(2x)-\cos(2x)} = \frac{1+2\sin(x)\cos(x)+\cos(2x)}{1+2\sin(x)\cos(x)-\cos(2x)}$$

$$= \frac{1+2\sin(x)\cos(x)+\cos(2x)}{1+2\sin(x)\cos(x)-\cos(2x)} = \frac{1+2\sin(x)\cos(x)+(2\cos^2(x)-1)}{1+2\sin(x)\cos(x)-(1-2\sin^2(x))}$$

$$= \frac{1-1+2\sin(x)\cos(x)+2\cos^2(x)}{1-1+2\sin(x)\cos(x)+2\sin^2(x)}$$

$$= \frac{2\cos(x)\big(\sin(x)+\cos(x)\big)}{2\sin(x)\big(\cos(x)+\sin(x)\big)}$$

$$= \frac{\cos(x)}{\sin(x)} = \cot(x)$$

33. $(-4\sin(x)\cos(x)+2\cos(2x))^2+(2\cos(2x)+4\sin(x)\cos(x))^2$

$$= (-2\cdot 2\sin(x)\cos(x)+2\cos(2x))^2+(2\cos(2x)+2\cdot 2\sin(x)\cos(x))^2$$

Use the double angle formula for sine ($2\sin(x)\cos(x)=\sin(2x)$):

$$= (-2\cdot \sin(2x)+2\cos(2x))^2+(2\cos(2x)+2\cdot \sin(2x))^2$$

Now we have to square these terms out:

$$= 4\sin^2(2x)-8\sin(2x)\cos(2x)+4\cos^2(2x)+4\cos^2(2x)+8\sin(2x)\cos(2x)+4\sin^2(2x)$$

Combine like terms:

$$= 8\sin^2(2x)+8\cos^2(2x)+8\sin(2x)\cos(2x)-8\sin(2x)\cos(2x)$$

Clean up a little more:

$$= 8(\sin^2(2x)+\cos^2(2x))+0$$

Apply THE Pythagorean identity ($\sin^2(?)+\cos^2(?)=1$):

$$= 8(1) = 8$$