MTH 112 HW 6.3 Winter 2009

1. $\sec(x) - \sin(x)\tan(x) = \cos(x)$
   Rewrite left side with sine and cosine, and multiply both sides by cosine:

$$\cos(x)\left(\frac{1}{\cos(x)} - \sin(x)\frac{\sin(x)}{\cos(x)}\right) = \cos(x)\cos(x)$$

   Distribute and reduce:

$$1 - \sin^2(x) = \cos^2(x)$$

   This is a version of THE Pythagorean identity.

2. $\dfrac{1+\cos(\theta)}{\sin(\theta)} + \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{\cos(\theta)+1}{\sin(\theta)\cos(\theta)}$
   Multiply both sides by GCD:

$$\sin(\theta)\cos(\theta)\left(\frac{1+\cos(\theta)}{\sin(\theta)} + \frac{\sin(\theta)}{\cos(\theta)}\right) = \sin(\theta)\cos(\theta)\left(\frac{\cos(\theta)+1}{\sin(\theta)\cos(\theta)}\right)$$

   Distribute and cancel:

$$\cos(\theta)\big(1+\cos(\theta)\big) + \sin(\theta)\sin(\theta) = \cos(\theta)+1$$

   Continue to simplify:

$$\cos(\theta) + \cos^2(\theta) + \sin^2(\theta) = \cos(\theta)+1$$

   Apply Pythagoren Identity:

$$\cos(\theta)+1 = \cos(\theta)+1$$

   Boom, Done!

3. $\dfrac{1-\cos(x)}{\sin(x)} = \dfrac{\sin(x)}{1+\cos(x)}$
   Cross multiply:

$$\big(1-\cos(x)\big)\big(1+\cos(x)\big) = \sin(x)\sin(x)$$

   Distribute:

$$1 - \cos^2(x) = \sin^2(x)$$

   This is a version of THE Pythagorean identity.

4. $\dfrac{1+\tan(y)}{1+\cot(y)} = \dfrac{\sec(y)}{\csc(y)}$
   Cross multiply:

5. $\dfrac{1+\tan(\theta)}{1-\tan(\theta)} + \dfrac{1+\cot(\theta)}{1-\cot(\theta)} = 0$

6. $\dfrac{\sin(x)+\cos(x)}{\sec(x)+\csc(x)} = \dfrac{\sin(x)}{\sec(x)}$

7. $\dfrac{\cos^2(\alpha) + \cot(\alpha)}{\cos^2(\alpha) - \cot(\alpha)} = \dfrac{\cos^2(\alpha)\tan(\alpha) + 1}{\cos^2(\alpha)\tan(\alpha) - 1}$

8. $\sec(2\theta) = \dfrac{\sec^2(\theta)}{2 - \sec^2(\theta)}$

9. $\dfrac{2\tan(\theta)}{1 + \tan^2(\theta)} = \sin(2\theta)$

10. $\dfrac{\cos(u - v)}{\cos(u)\sin(v)} = \tan(u) + \cot(v)$