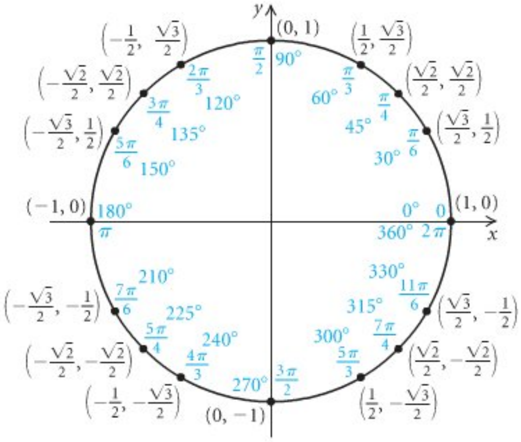
MTH 112 HW 6.4 Winter 2009

1, 3, 13, 21, 37, 39, 41, 45, 51, and 55

To do most of the problems in section 4 you will have to draw a triangle and/or use the unit circle.



1. $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

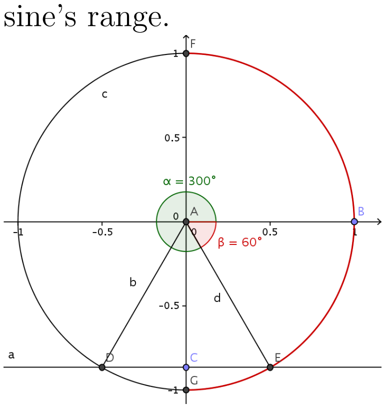
   First, an inverse trig function produces an angle so,

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \theta$$

   or,

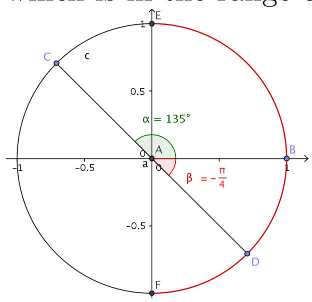
$$\sin(\theta) = \frac{-\sqrt{3}}{2}$$

   So, for what angles on the unit circle is the y-value equal to $\frac{-\sqrt{3}}{2}$?
   That would be at $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$. Note that $\frac{5\pi}{3}$ puts us in the same place as $-\frac{\pi}{3}$, which is in the arc sine's range.



   Alternatively, just type it into your calculator. If you are in degree mode, then you should get $^{-}60°$.

3. $\tan^{-1}(-1)$
Remember, $\tan^{-1}(-1) = \theta$. This means $\tan(\theta) = -1$. For what angle(s), does the tangent equal -1, i.e., the slope is -1. Then $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ and variations of these like $-\frac{\pi}{4}$ or $-45°$, which is in the range of the arc tan.



13. $\cot^{-1}\left(-\sqrt{3}\right)$
The $\cot^{-1}\left(-\sqrt{3}\right)$ is an angle, so rewrite it as follows:

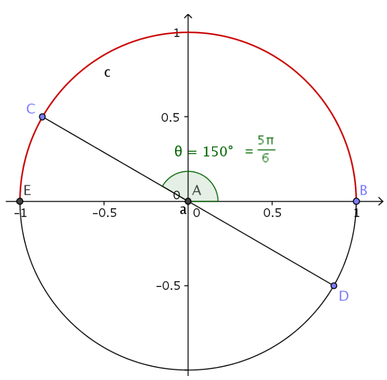
$$\cot^{-1}\left(-\sqrt{3}\right) = \theta$$

or

$$\cot(\theta) = -\sqrt{3} \; or \; \tan(\theta) = -\frac{1}{\sqrt{3}}$$

So, θ should be between 0 and π for the arc cotangent.

$$\theta = \frac{5\pi}{6} rad = 150°$$



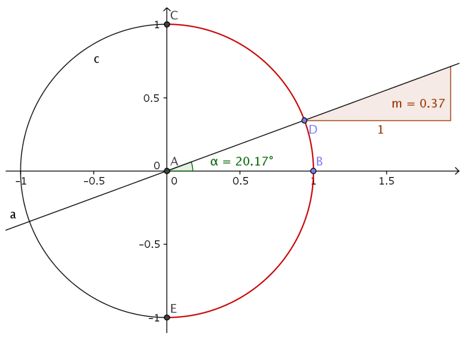
Caution here: If you view $\cot^{-1}(x)$ as $\tan^{-1}\left(\frac{1}{x}\right)$ then the range will be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, excluding zero. This can be a problem, since:

$$\cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6} rad$$

$$\tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = -\frac{\pi}{6} rad$$

21. $\tan^{-1}(0.3673)$

$$\theta = \tan^{-1}(0.3673) \approx 0.3520\,rad \approx 20.2^\circ$$

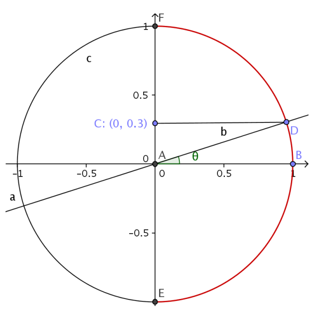


37. $\sin(\sin^{-1}(0.3))$
The $\sin^{-1}(0.3)$ is an angle, so rewrite it as follows:

$$\sin^{-1}(0.3) = \theta$$

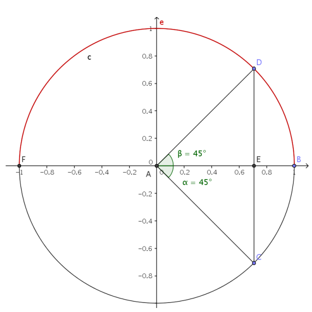
Now if we translate that to an equation with sine, then we're done:

$$\sin(\theta) = 0.3$$



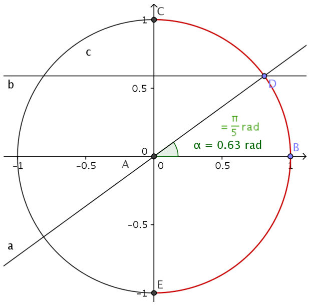
39. $\cos^{-1}\left[\cos\left(-\dfrac{\pi}{4}\right)\right]$
The arc cosine has a range from 0 to π, so:

$$\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right] = \cos^{-1}\left[\frac{\sqrt{2}}{2}\right] = \frac{\pi}{4}$$



41. $\sin^{-1}\left(\sin\left(\frac{\pi}{5}\right)\right)$
Since $\frac{\pi}{5}$ is in the range of the arcsine:

$$\sin^{-1}\left(\sin\left(\frac{\pi}{5}\right)\right) = \frac{\pi}{5}$$



45. $\sin\left[\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right]$
The $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ is an angle, so rewrite it as follows:

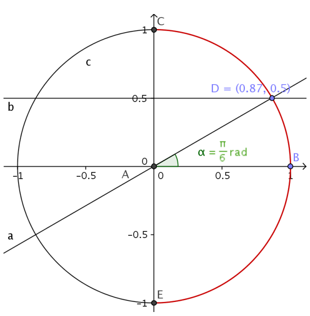
$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \theta$$

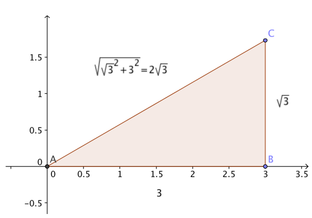
so,

$$\tan(\theta) = \frac{\sqrt{3}}{3}$$

and

$$\sin(\theta) = \frac{\sqrt{3}}{2}$$





51. $\tan(\sin^{-1}(0.1))$
Since $\sin^{-1}(0.1)$ is an angle, then rewrite it as follows:

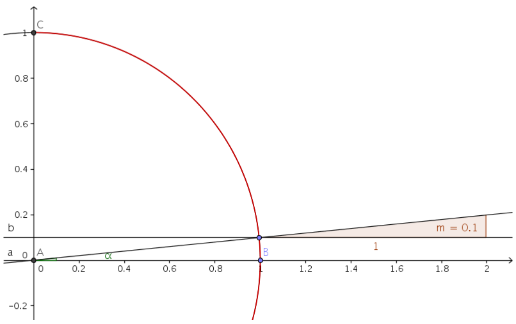
$$\theta = \sin^{-1}(0.1)$$

So,

$$\sin(\theta) = 0.1$$

and using the Pythagorean theorem we get

$$\tan(\theta) = \frac{0.1}{\sqrt{1 - 0.1^2}} \approx 0.10050378$$



55. $\sin\left(\tan^{-1}\left(\frac{a}{3}\right)\right)$
Since $\tan^{-1}\left(\frac{a}{3}\right)$ is an angle then rewrite as follows:

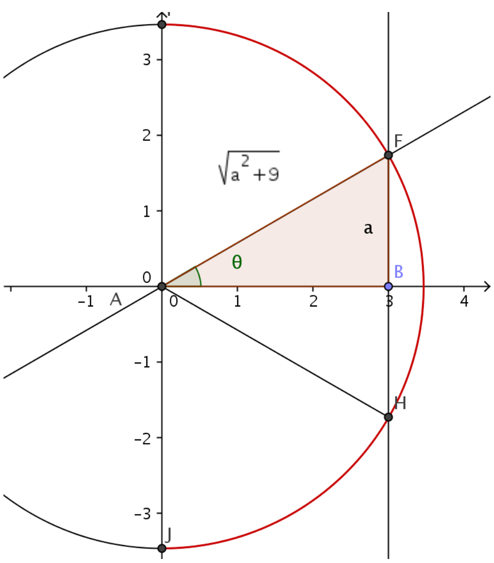
$$\theta = \tan^{-1}\left(\frac{a}{3}\right)$$

So,

$$\tan(\theta) = \frac{a}{3}$$

and applying the Pythagorean theorem we get

$$\sin(\theta) = \frac{a}{\sqrt{a^2 + 9}}$$



NOTE: a could be positive or negative, and so could the sine here.