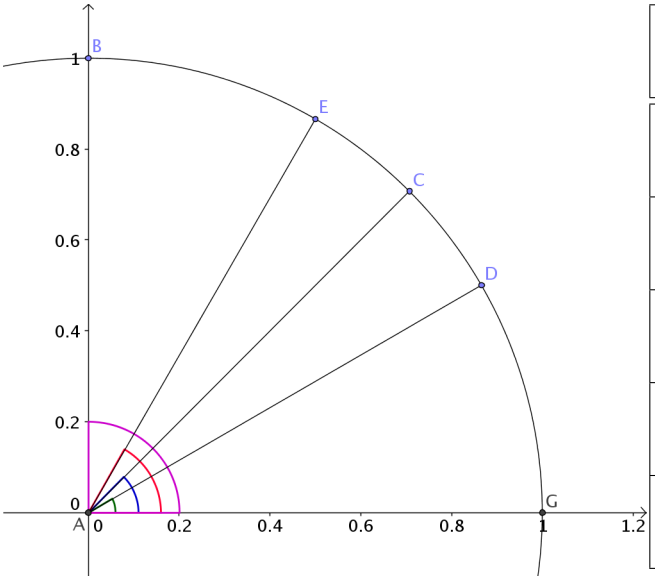
Math 112 Sample Final Winter 2009

Name ANSWERS Date _____

Please show ALL of your work if full or partial credit is desired. Communicating your solution is as important as stating your answer.

PLEASE, NO CALCULATORS ON THIS FIRST PAGE. THANK YOU.

1. Label the special angels and their x- and y- coordinates for the first quadrant and fill out the table.



θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$

2. Find the following exactly:

   (a) $\arcsin\left(\frac{1}{2}\right) =$

   (b) $\arccos\left(\frac{-1}{2}\right) =$

   (c) $\arctan\left(-1\right) =$

   (d) $\arctan\left(-\frac{1}{\sqrt{3}}\right) =$

3. Find $\cos\left(\arcsin\left(\frac{2}{3}\right)\right)$ exactly.

4. Rewrite the following:

   (a) $\sin^2(something) + \cos^2(something) =$

   (b) $\sin(-\theta) =$

   (c) $\cos(-\theta) =$

   (d) $\cos^2(\theta) =$

   (e) $1 - \sin^2(\theta) =$

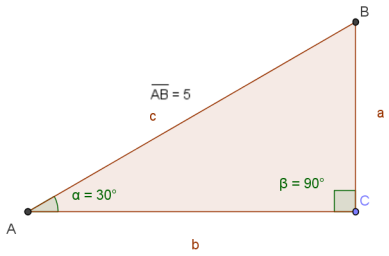
   (f) $\cos(\phi - \theta) =$

5. In a circle of radius 80 cm an arc of 251.33 cm long subtends an angle of how many radians? How many degrees? (answers to the nearest hundredth)

$$\theta = \frac{251.33}{80}\,\text{rad} \approx 3.14\,\text{rad} \approx 180.00^\circ$$

6. Determine the amplitude, period and phase shift (if any) of $y = 5\cos(3x - 2)$.

7. Solve the triangle, giving the missing sides and angles exactly. $A = \frac{\pi}{6} rad$, $c = 5\text{cm}$, $C = \frac{\pi}{2} rad$



$$\frac{5}{\sin(\frac{\pi}{2})} = \frac{a}{\sin(\frac{\pi}{6})}$$

$$\frac{5}{1} = \frac{a}{1/2}$$

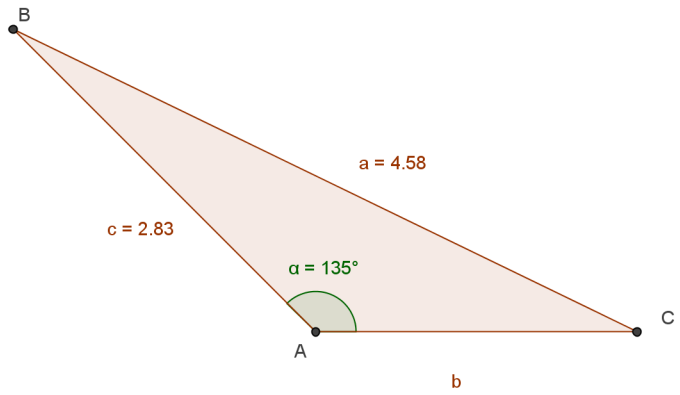
$$a = 5/2$$

$$(5/2)^2 + b^2 = 5^2$$

$$b = \sqrt{25 - 25/4} = \frac{\sqrt{75}}{2} = \frac{5\sqrt{3}}{2}$$

Alternatively, you could do this with proportions, noticing that this triangle is proportional to a 30-60-90 triangle with hypotenuse of 1.

8. Solve the triangle, giving the missing sides and angles to the nearest thousandth. $A = \frac{3\pi}{4} rad$, $c = 2\sqrt{2}\,\text{cm}$, $a = \sqrt{21}$



$$\frac{\sin(\frac{3\pi}{4})}{\sqrt{21}} = \frac{\sin(C)}{2\sqrt{2}}$$

$$\frac{\sqrt{2}/2}{\sqrt{21}} = \frac{\sin(C)}{2\sqrt{2}}$$

$$\sin(C) = 2\sqrt{2}\frac{\sqrt{2}/2}{\sqrt{21}}$$

$$\sin(C) = \frac{2}{\sqrt{21}}$$

$$\arcsin(\frac{2}{\sqrt{21}}) \approx 25.876690061$$

and

$$B \approx 180 - 135 - 25.876690061 = 19.1230994$$

and

$$\frac{b}{\sin(19.1230994)} = \frac{\sqrt{21}}{\sin(\frac{3\pi}{4})}$$

$$b \approx 2.1230830829$$

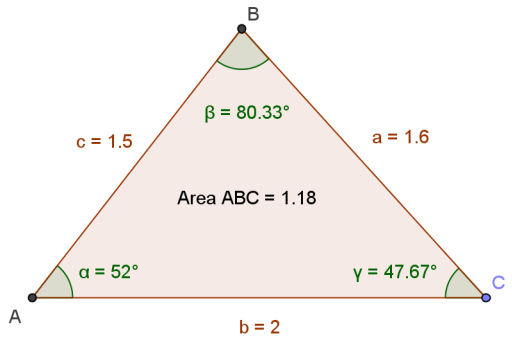
9. Solve the triangle, giving the missing sides and angles to the nearest thousandth. $A = 52°$, $b = 2\text{cm}$, $c = 1.5$

$$a = \sqrt{2^2 + 1.5^2 - 2 \cdot 2 \cdot 1.5 \cdot \cos(52°)} \approx 1.5987592527$$

$$\frac{\sin(B)}{2} = \frac{\sin(52°)}{1.5987592527}$$

$$B \approx \arcsin(0.9857779) \approx 80.325337°$$

$$C \approx 180 - 52 - 80.325337 = 47.674663°$$



10. Find the area of the triangle from the previous problem.

$$A = 0.5 \cdot 2 \cdot 1.5 \cdot \sin(52°) \approx 1.1820161\,\text{cm}^2$$

11. Find the exact values of:

    (a) $\sin(210°)$

    (b) $\tan\left(\frac{5\pi}{4}\right)$

    (c) $\cos\left(-\frac{2\pi}{3}\right)$

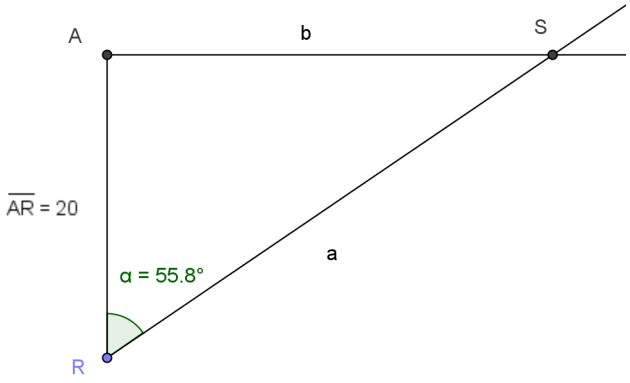
12. Use a reference angle to find the exact value of $\cos\left(\frac{7\pi}{4}\right)$. Do not use a calculator. Draw a picture on a unit circle.

13. The $\cos(\theta) = -\frac{1}{5}$ in quadrant III. Find the EXACT values of the other five functions.

14. The $\cos(\theta) = -\frac{6}{7}$ in quadrant II. Find the value of θ to the nearest hundredth of a radian.

15. An amplifying tower is situated due west of a radio station. A receiver is 20 km south of the amplifying tower. From the receiver, the bearing to the station is N55°48′E. How far is the amplifying tower from the station to the nearest meter?

    First draw a picture:



    We need to know length b in my picture, which is the opposite side from the angle given. So,
    $\tan(55.8°) = \dfrac{b}{20\,\text{km}}$
    $b = 20\,\text{km} \cdot \tan(55.8°) \approx 29.429\,\text{km}$

16. Gary's bicycle has wheels with a 650mm diameter. If the wheel is making 3 revolutions per second, what is the speed of Gary's bicycle to the nearest kilometer per hour?

$$\frac{3\,\text{rev}}{1\,\text{sec}} \cdot \frac{\pi \cdot 650\,\text{mm}}{1\,\text{rev}} \cdot \frac{1\,\text{m}}{1000\,\text{mm}} \cdot \frac{1\,\text{km}}{1000\,\text{m}} \cdot \frac{3600\,\text{sec}}{1\,\text{hr}} \approx 22\,\text{km/hr}$$

17. Graph the function $f(x) = \sin(x)$. Identify the period, amplitude, and phase shift. Show 2 periods. Label the x-intercepts, the y-intercept, and the maximum and minimum values exactly.

18. Graph the function $f(x) = 2\sin(\pi x) + 1$. Identify the period, amplitude, and phase shift. Show 2 periods. Label the x-intercepts, the y-intercept, and the maximum and minimum values exactly.

19. Graph $\theta = \arccos(x)$. Include a table of at least 6 points.

20. Express $\tan(\arccos(x))$ in terms of x without trigonometric functions. Hint: draw a triangle.

21. Show that $\frac{\sin(\theta)}{\sin(\pi/2-\theta)} = \frac{opposite}{adjacent}$ in any right triangle.

22. Show that $\sin(2x) + \cos(2x) = 2\left(\cos(x) - \sin(x)\right)\left(\cos(x) + \sin(x)\right)$ for all values of x.

$$2\left(\cos(x) - \sin(x)\right)\left(\cos(x) + \sin(x)\right) = 2\left(\cos^2(x) - \sin^2(x)\right) = 2\cos(2x)$$

So,

$$\sin(2x) + \cos(2x) = 2\cos(2x)$$

Subtract $\cos(2x)$ from each side,

$$\sin(2x) = \cos(2x)$$

Uh, oh! These aren't equal unless $x = \frac{pi}{8} + \frac{\pi}{2}n$

23. Find all values of θ such that $\cos(\theta) = -\frac{1}{2}$.

24. Find all solutions to $\cos(2\theta) = 2\cos(\theta)$ on the interval $[-2\pi, 2\pi]$

$$\cos(2\theta) = 2\cos(\theta)$$

$$2\cos^2(\theta) - 1 = 2\cos(\theta)$$

$$2\cos^2(\theta) - 2\cos(\theta) - 1 = 0$$

Let $u = \cos(\theta)$,

$$2u^2 - 2u = 1$$

$$u = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

So,

$$\cos(\theta) = \frac{1 + \sqrt{3}}{2} \approx 1.37$$

OR

$$\cos(\theta) = \frac{1 - \sqrt{3}}{2} \approx -0.37$$

but $\cos(\theta) \leq 1$ so,

$$\theta = arccos\left(\frac{1 - \sqrt{3}}{2}\right) \approx 1.9455 \text{ rad}$$

OR -1.9455, $2\pi - 1.9455$, $1.9455 - 2\pi$

25. Find the exact value of the $\sin\left(\frac{\pi}{12}\right)$ in two different ways.

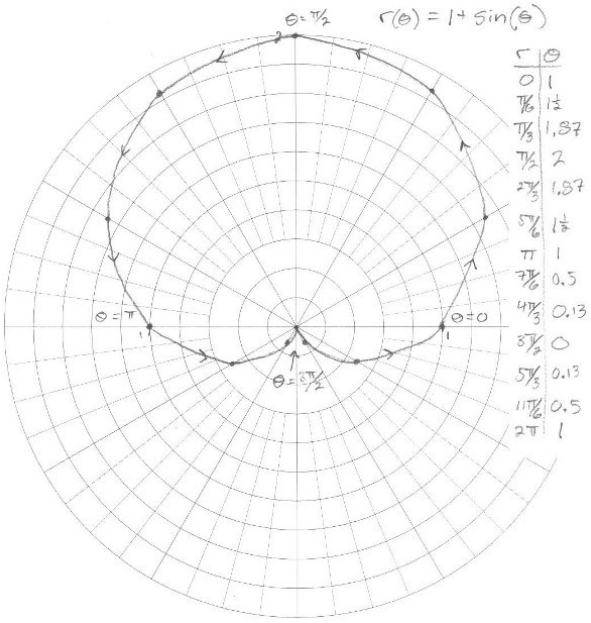
$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\frac{\pi}{6}}{2}\right) =$$

OR

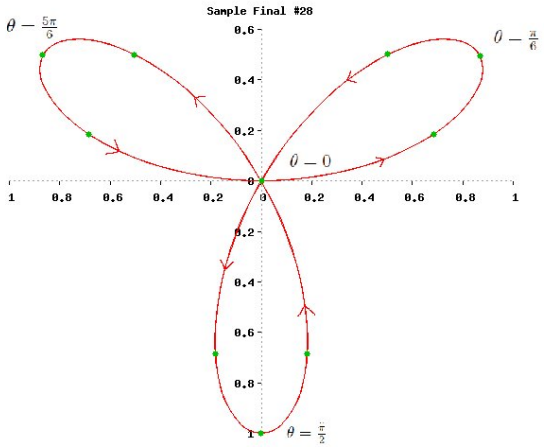
$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) =$$

26. Try #26 in section 7.2.

27. Graph the polar function $r(\theta) = 1 + \sin(\theta)$. Include a table showing at least 8 points, show the direction of the graph and label 4 values of θ on the curve.



28. Graph the polar function $r(\theta) = \sin(3\theta)$. Include a table showing at least 8 points, show the direction of the graph and label 4 values of θ on the curve.



29. A force of 15 pounds and a force of 20 pounds act on an object at right angles to each other. Find the sum, or resultant, giving the magnitude of the resultant and the angle that it makes with the larger force.

30. Vector $\vec{v}$ is 3 cm long E60°N. Vector $\vec{u}$ is 2 cm long E30°N. They can be used to make a parallelogram. Draw the parallelogram. Find the lengths of the diagonals of the parallelogram.

31. Convert the polar coordinates to rectangular coordiantes.

    (a) $(5; 45°)$

    (b) $(-1; -60°)$

32. Convert rectangular coordinates to polar coordiantes.

    (a) $(11, 7)$

(b) $(-1.2, 1.6)$