

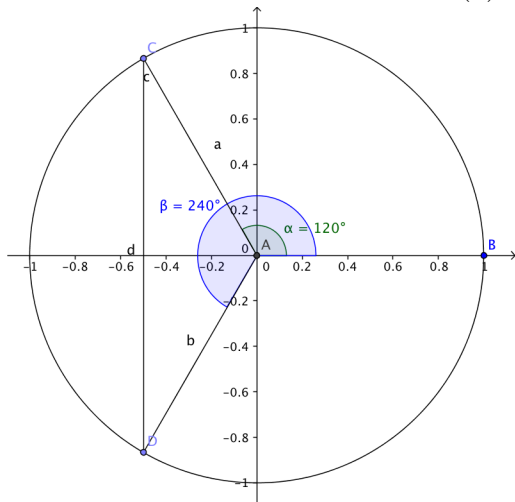
Name _____

Date _____

Please show ALL of your work if full or partial credit is desired. Communicating your solution is as important as stating your answer.

PLEASE, NO CALCULATORS ON THIS FIRST PAGE. THANK YOU.

1. Find all values of θ such that $\cos(\theta) = -\frac{1}{2}$.



$$\theta = \frac{2\pi}{3} + 2\pi n \text{ or } \frac{4\pi}{3} + 2\pi n, \text{ where } n \text{ is an integer}$$

2. Find $\cos\left(\arcsin\left(\frac{5}{13}\right)\right)$ exactly.

$$\cos\left(\arcsin\left(\frac{5}{13}\right)\right) = \frac{12}{13}$$

3. If $\theta = \frac{4\pi}{3}$, then find the following exactly:

(a) $\arcsin(\sin(\theta)) = \frac{-\pi}{3}$

(b) $\arccos(\cos(\theta)) = \frac{2\pi}{3}$

(c) $\arctan(\tan(\theta)) = \frac{\pi}{3}$

Rewrite the following:

4. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$

7. $\sec^2(\theta) - 1 = \tan(\theta)$

5. $\sin(-\theta) = -\sin(\theta)$

8. $\cos(\theta - \phi) = \cos(\theta)\cos(\phi) + \sin(\theta)\sin(\phi)$

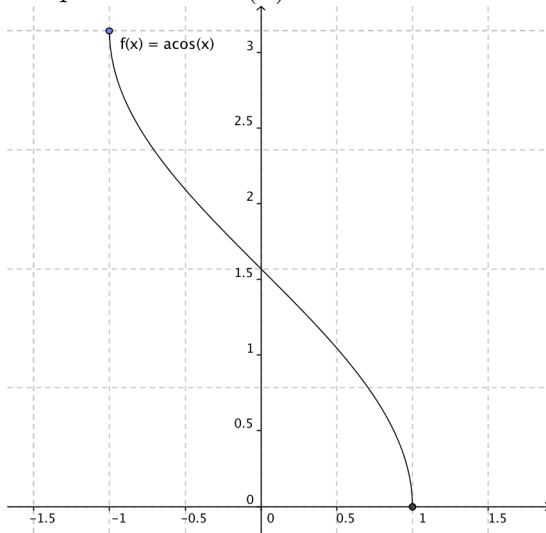
6. $1 - \cos^2(\theta) = \sin^2(\theta)$

9. $2\sin(\theta)\cos(\theta) = \sin(2\theta)$

10. Simplify: $\sin^2(3\theta) + \cos^2(3\theta)$

$$\sin^2(3\theta) + \cos^2(3\theta) = 1$$

11. Graph $\theta = \arccos(x)$. Include a table of values.



12. Express $\tan(\arccos(x + 1))$ in terms of x without trigonometric functions.
BONUS: For what values of x is the expression above defined?

$$\tan(\arccos(x + 1)) = \sqrt{1 - (x + 1)^2}, \quad x \in [-2, -1) \cup (-1, 0]$$

13. Show that $2 \cos(\theta) \sin(\phi) = \sin(\theta + \phi) - \sin(\theta - \phi)$.

$$\begin{aligned} \sin(\theta + \phi) - \sin(\theta - \phi) &= \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) - \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) \\ &= \sin(\theta) \cos(\phi) - \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) + \sin(\phi) \cos(\theta) \\ &= 0 + 2 \cos(\theta) \sin(\phi) \end{aligned}$$

14. Show that

$$\frac{1 + \sin(x)}{\cos(x)} = \frac{\cos(x)}{1 - \sin(x)}$$

Cross multiply:

$$(1 + \sin(x))(1 - \sin(x)) = \cos(x) \cos(x)$$

Carry out the multiplication:

$$1 - \sin^2(x) = \cos^2(x)$$

Add $\sin^2(x)$ to each side:

$$1 = \sin^2(x) + \cos^2(x)$$

$$1 = 1$$

Boom, Done!

15. Find all solutions to $\sin(2\theta) = 2\sin(\theta)$ on the interval $[-2\pi, 2\pi]$

$$2\sin(\theta)\cos(\theta) = 2\sin(\theta)$$

$$2\sin(\theta)\cos(\theta) - 2\sin(\theta) = 0$$

$$2\sin(\theta)(\cos(\theta) - 1) = 0$$

$$2\sin(\theta) = 0 \text{ OR } \cos(\theta) - 1 = 0$$

$$\sin(\theta) = 0 \text{ OR } \cos(\theta) = 1$$

$$\theta = -2\pi, -\pi, 0, \pi, 2\pi \text{ OR } \theta = -2\pi, 0, 2\pi$$

16. Find the exact value of the $\sin\left(\frac{\pi}{12}\right)$.

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\frac{\pi}{6}}{2}\right) = \sqrt{\frac{1 - \cos\left(\frac{\pi}{6}\right)}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$$

Math 212 Chapter 5 Take-Home Exam Spring 2004

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Take-Home Exam Questions

1.

2.